Petri Nets

1. **Basic Petri Net Model**
2. **Properties and Analysis of Petri Nets**
3. **Extended Petri Net Models**

Petri Nets

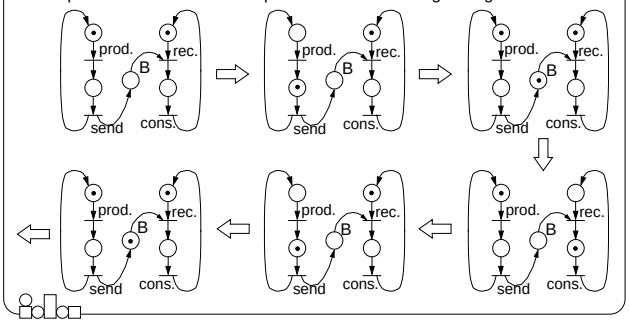
- Systems are specified as a directed bipartite graph.
  The two kinds of nodes in the graph:
  1. *Places*: they hold the distributed state of the system expressed by the presence/absence of tokens in the places.
  2. *Transitions*: denote the activity in the system
- *The state of the system*: captured by the marking of the places (number of tokens in each place)

Petri Nets (cont'd)

- The dynamic evolution of the system: determined by the firing process of transitions.
  - A transition may fire whenever all its predecessor places are marked.
  - If a transition fires it removes a token from each predecessor place and adds a token to each successor place.

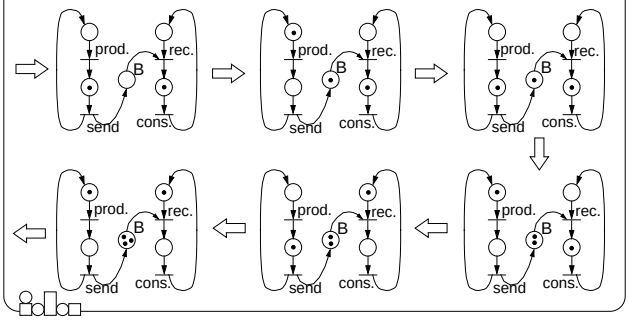
Petri Net Example

A producer and a consumer process communicating through a buffer:



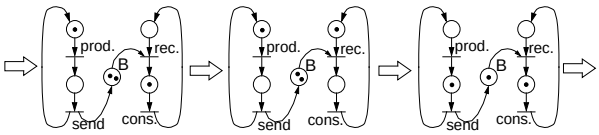
Petri Net Example (cont'd)

Continuation from previous slide:



Petri Net Example (cont'd)

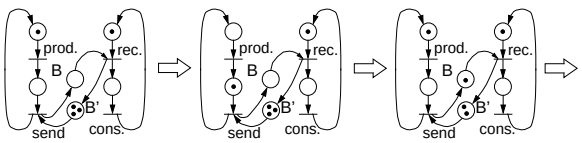
Continuation from previous slide:



☞ Notice that the buffer is considered to be infinite (tokens can accumulate in place *B*).

Petri Net Example (cont'd)

Here we have the same model as on the previous slides, but with limited buffer. The buffer size is three (number of initial tokens in *B*')



Total number of tokens in *B* and *B*' is constant.

Some Features and Applications of Petri Nets

- Intuitive.
  Easy to express concurrency, synchronisation, nondeterminism.

  Nondeterminism is an important difference between Petri nets and dataflow!

- As an uninterpreted model, Petri Nets can be used for several, very different classes of problems.

  - *Uninterpreted model*: nothing has to be specified related to the particular activities associated to the transitions.

Some Features and Applications of Petri Nets (cont'd)

- Petri Nets have been intensively used for modeling and analysis of industrial production systems, information systems, but also
  - Computer architectures
  - Operating systems
  - Concurrent programs
  - Distributed systems
  - Hardware systems

Properties and Analysis of Petri Nets

☞ Several properties of the system can be analysed using Petri nets:

- *Boundedness*: the number of tokens in a certain place does not exceed a given limit.
  If this limit is 1, the property is sometimes called *safeness*.
  - You can check that available resources are not exceeded.

Properties and Analysis of Petri Nets (cont'd)

- *Liveness*:
  - A transition *t* is called live if for every possible marking there exists a chance for that transition to become enabled.

  The whole net is live, if all its transitions are live.
  - liveness is important in order to check that a system is not deadlocked.
- *Reachability*: given a current marking M of the net, and another marking M', does there exist a sequence of transitions by which M' can be obtained?
  - You can check that a certain desired state (marking) is reached.
  - You can check that a certain undesired state is never reached.

Properties and Analysis of Petri Nets (cont'd)

Mathematical tools are available for analysis of Petri Nets.

The properties discussed above can be formally verified.

☞ Petri nets (like dataflow systems) are *asynchronous concurrent*.

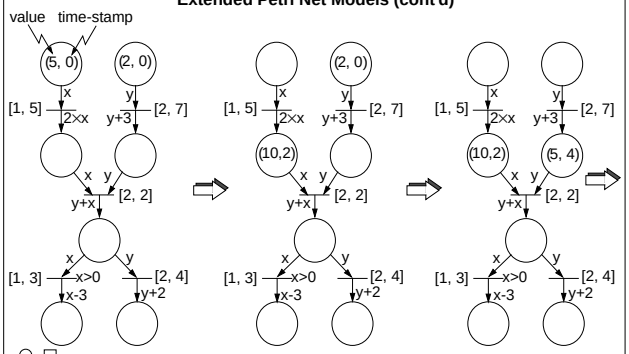
- Events can happen at any time.
- There exists a partial order of events:

Extended Petri Net Models

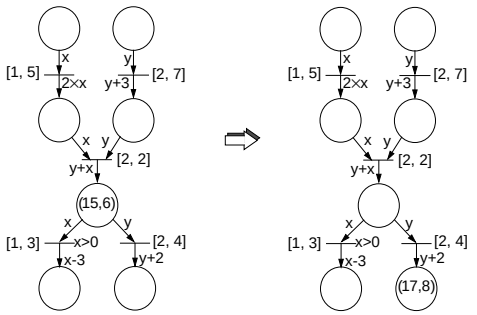
Basic Petri Net models have a limited expressive power.

- Timed Petri Nets
  - Transitions have associated times (time intervals)
  - Tokens are carrying time stamps.
  - With timed Petri nets we can model the timing aspects
- Coloured Petri Nets
  - Tokens have associated values
  - Transitions have associated functions
  - Coloured Petri Nets are similar to dataflow models (but also capture nondeterminism!).

Extended Petri Net Models (cont'd)



Extended Petri Net Models (cont'd)



Extended Petri Net Models (cont'd)

- Extended Petri Nets have a larger expressive power then classical Petri Nets.

⇩

Analysis is more complex; the formal analysis of properties can take unacceptably large amounts of time (memory).

- Simulation of the Petri Net can be used in order to verify the system and to estimate performance

Summary

- Petri Nets are a mixture of dataflow and state-based model. *Places* hold the distributed state of the system (represented by the marking); *transitions* denote the activity of the system.
- Petri nets elegantly capture concurrency, synchronisation, and nondeterminism.
- A large class of problems can be solved using Petri Net modelling; system properties like boundedness, liveness, and reachability can be formally analysed.
- Petri nets, like dataflow, are asynchronous, concurrent models. Events can happen at any time; there exists a partial order of events.

Summary (cont'd)

- Basic Petri Nets are limited in their expressive power
  - in timed Petri Nets an explicit notion of time has been introduced;
  - in coloured Petri nets tokens have associated values.
- Formal reasoning about extended Petri Net models is very difficult because of complexity issues.
  Simulation of the models is often used for system validation.

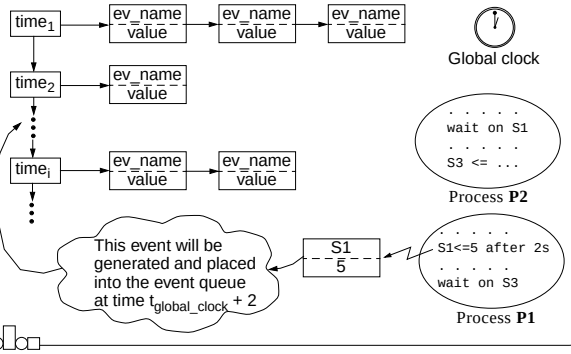
Discrete Event Models

1. **What Is a Dicrete Event Model?**
2. **Disctere Event Simulation**
3. **Efficiency of Discrete Event Simulation**
4. **Potential Ambiguities in Discrete Event Simulation**

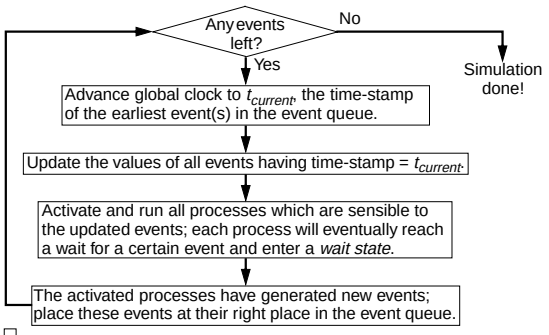
Discrete Event Models

- The system is a collection of processes that respond to events.
- *Each event carries a time-stamp indicating the time at which the event occurs.*
- *Time-stamps are totally ordered.*
- A Discrete Event (DE) simulator maintains a *global event queue sorted by the time-stamps.* The simulator also keeps a single global time.

Discrete Event Simulator



Discrete Event Simulator (cont'd)



Discrete Event Simulation

- The discrete event model has been mainly used for system simulation.
  - Several languages have been developed for system modeling based on the discrete event model. Most well known:
    - VHDL, Verilog (both used for hardware modeling)
- Efficient way to simulate distributed systems.

  In general, efficient for large systems with autonomous components, with relatively large idle times. Systems with non-regular, possibly long times between different activities.

  Why is this the case?

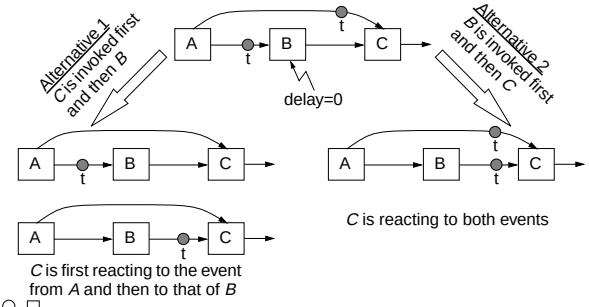
  Because DE simulation will only consider the particular times when a change in the system (an event) occurs. This is opposed to, for example, cycle-based models, where *all clock-ticks* are considered.

Discrete Event Simulation (cont'd)

- Efficiency related problems:
  - Keeping the sorted event-queue is time-consuming.
  - As the activity of the simulated system increases (a lot of events at a very high number of time moments have to be considered) the overhead becomes high ⇒ simulation is slow.
- Event driven models are primarily employed for simulation.
  - Functional verification
  - Performance evaluation
- Both synthesis and formal verification are very difficult (complex) with DE models.
  - The classical trade-off between expressive power and the possibility of formal reasoning and efficient synthesis.

A Problem with Discrete Event Simulation

Simultaneous events resulted from 0 delay components:



A Problem with Discrete Event Simulation (cont'd)

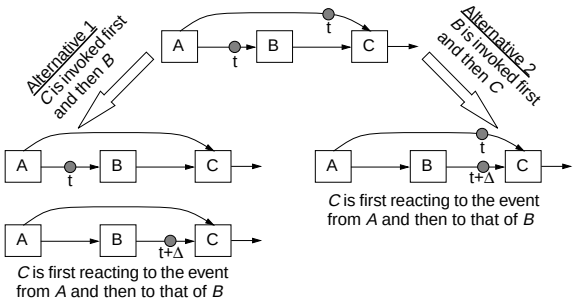
Such an ambiguity creates problems. For example, different simulators will produce different output for the same model with identical inputs.

☞ A possible solution (used, for example, in VHDL):
Each instant of "real time" is virtually broken into a potentially infinite number of totally ordered "delta steps".

⇩

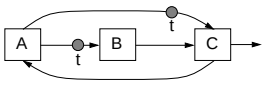
A zero delay process will introduce a "delta delay" on the event.
The input and output event are at the same "real time", but a partial order is introduced.

A Problem with Discrete Event Simulation (cont'd)



A Problem with Discrete Event Simulation (cont'd)

☞ This also solves the problem of *zero-delay feedback loops.*
(delay on all three components is 0, see also FSMs, F6 6 - slide 18)



Not only non-determinism is a problem here, but the output on *C* can, in general, not be determined.

Some systems reject such models.

If delta delays are introduced (like with VHDL) a deterministic simulation is possible.

Summary

- Discrete Event is a very powerful modelling technique, in terms of expressive power.
  Models are concurrent and asynchronous.
  The timing model is also very general. Delays on computation or communication can be arbitrary.
- We pay for the large expressive power by the reduced potential of formal reasoning and efficient synthesis.
  Discrete event models are mainly used for simulation.
- Simulation is based on maintaining a unique, sorted event queue. This can create problems with simulation efficiency for large system models.
- In order to avoid problems with zero delay computations, “delta delays” are used (e.g. in VHDL).